58. Rudenko, N. S., *J. Exp. Theoret. Phys.*, 9, 1078 (1939).
59. ———, and L. W. Shubnikov, *Physik. Z. Sowjetunion*, **6**, 470 (1934).
60. Sage, B. H., and W. N. Lacey, *Ind. Eng. Chem.*, **30**, 829 (1938).
61. Sage, B. H., J. G. Schaafsma, and W. N. Lacey, *ibid.*, **26**, 1218 (1934).
62. Sage, B. H., D. C. Webster, and W. N. Lacey, *ibid.*, **29**, 658 (1937).
63. *Ibid.*, p. 1188.
64. Sage, B. H., W. D. Yale, and W. N. Lacey, *ibid.*, **31**, 223 (1939).
65. Schaefer, C. A., and George Thodos, *A.I.Ch.E. Journal*, **5**, 155 (1959).
66. *Ibid.*, p. 367.
67. Schiller, W., *Forsch. Gebiete Ingenieurw.*, **5**, 71 (1934).
68. Shimotake, Hiroshi, and George Thodos, *A.I.Ch.E. Journal*, **4**, 257 (1958).
69. Shimotake, Hiroshi, Ph.D. dissertation, Northwestern University, Evanston, Illinois (1960).
70. Shugayev, W., *Physik. Z. Sowjetunion*, **5**, 659 (1934).
71. Sigwart, K., *Forsch. Gebiete Ingenieurw.*, **7**, 125 (1936).
72. Smith, A. S., and G. G. Brown, *Ind. Eng. Chem.*, **35**, 705 (1943).
73. Stakelbeck, H., *Z. ges. Kälte Ind.*, **40**, 33 (1933).
74. Stiel, L. I., and George Thodos, *A.I.Ch.E. Journal*, **7**, 611 (1961).
75. Stiel, L. I., M. S. thesis, Northwestern University, Evanston, Illinois (1960).
76. Swift, G. W., Ph.D. dissertation, University of Kansas, Lawrence, Kansas (1960).
77. Theiss, R. V., M.S. thesis, Northwestern University, Evanston, Illinois (1960).
78. Timroth, D., *J. Physics (U.S.S.R.)*, **2**, 149 (1940).
79. Uyehara, O. A., and K. M. Watson, *Natl. Petrol. News, Tech. Sec.*, **36**, R764 (October 4, 1944).
80. van Itterbeek, A., and O. van Paemel, *Physica*, **8**, 133 (1941).
81. Yee, E. G., Ph.D. dissertation, University of Michigan, Ann Arbor, Michigan (1936).

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A Note on Latent Heat Transport in Nucleate Boiling

S. G. BANKOFF
Northwestern University, Evanston, Illinois

A simple one-dimensional transport calculation is made which indicates that latent heat transport, by simultaneous evaporation and condensation at different portions of the bubble surfaces, can account for the major portion of the total heat flux in the neighborhood of the departure from subcooled nucleate boiling (burnout). This differs from the previously held view that the stirring action of the bubbles accounted for most of the heat flow in subcooled nucleate boiling. Other evidence is discussed which favors the latent heat transport theory.

Until recently it seemed quite probable that latent heat transport played a minor role in nucleate boiling heat transfer. This belief was based on observations of the rate of vapor production in subcooled nucleate boiling of water at atmospheric pressure (*1* to *3*). These measurements indicated that only 1 to 2% of the total heat flux could be accounted for by the latent heat content of the vapor evolved at the surface. Tentatively it was concluded that the bubbles acted principally as stirring devices which increased the turbulent intensity within the boundary layer next to the heating surface. This would imply that it would make no difference whether the bubbles were filled with vapor or with inert gas. Indeed it has been shown (*4*) that nonboiling heat transfer coefficients are increased by a factor of 2 or 3 by generating inert gas bubbles at the heating surface, although the resulting heat fluxes, even at the highest generation rate of inert gas bubbles, were in the range of the lower limit of nucleate boiling.

This leads to the suspicion that some other mechanism, such as transport by net flow of vapor away from the heating surface, may dominate near the upper limit of nucleate boiling. For this to be true there would have to be significant transport by simultaneous vaporization near the face of the bubbles and condensation at the top. This further implies that the heat transfer coefficients at the surface of the steam bubbles must be considerably greater than those calculated by Gunther and Kreith (*2*), who assumed that the bubbles were surrounded by a stationary film of liquid. The requirements for latent heat transport to be significant were discussed by Snyder and by Bankoff (*20*).

Direct experimental evidence concerning these points has been quite difficult to obtain because of the statistical nature of the boiling transfer and because of the small time and length scales for the temperature variations in the liquid. Recently however some direct measurements of the heat transfer coefficient at the surface of a rapidly growing and collapsing steam bubble in a turbulent subcooled liquid stream (*5*) have been presented. These coefficients ranged from 13,000 to the phenomenally high figure of over 300,000 B.t.u./hr./sq. ft./°F. and indicate that at least several maximum bubble volumes of steam flow through the bubble during its lifetime. From these measurements it was shown that latent heat transport becomes the dominant mechanism near the upper limit of nucleate boiling in the subcooled nucleate boiling experiments of water at atmospheric pressure performed by Gunther (*1*).

The purpose of this note is to point out, apart from this direct evidence, several other pieces of indirect evidence and to present some simple calculations which indicate that latent heat transport dominates at high fluxes. This means that as the bubbles become closely packed, the principal heat transfer into the bulk stream occurs from the bubble surfaces, rather than from the portions of the solid surface between the bubbles. This seems quite reasonable on the basis of physical intuition.

Some significant results can be obtained simply by examining the shape of the curves of a maximum heat flux in nucleate boiling vs. reduced pressure as reported for water (*6*) and several organic liquids (*7*). These

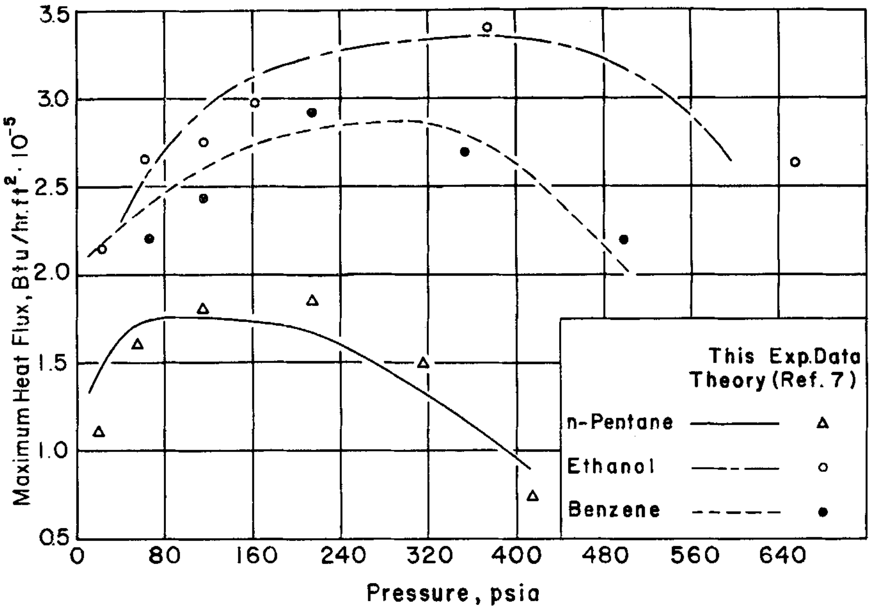

Fig. 1. Maximum heat flux in pool boiling: data of Cichelli and Bonilla (7) compared with Equations (1) and (2). Scale factor $F\epsilon$ for benzene, 5.2×10^{-3}; ethanol, 2.4×10^{-3}; n-pentane, 5.3×10^{-3}.

curves show a maximum at a reduced pressure in the neighborhood of 0.3 to 0.5. The maximum heat flux in nucleate boiling (popularly called *burnout*) represents the existence of a critical packing fraction of bubbles on the heating surface, such that coalescence of neighboring bubbles becomes probable. The mean turbulent intensity induced by the bubbles in the liquid between them has been calculated for an assembly of bubbles growing and collapsing on a surface in subcooled nucleate boiling (*8*). The resulting expression for the rms velocity induced by the bubbles u'_b is

$$u'_b = \dot{R}_a \sqrt{\frac{2F}{1-F}} \qquad (1)$$

This quantity may be identified with the mean turbulent intensity of the liquid between the bubbles and hence with the stirring action of the bubbles. At the upper limit of nucleate boiling the critical packing fraction F_c is of the order of 0.5. Hence the stirring effectiveness depends primarily upon the mean wall velocity of the bubbles and only weakly on the critical packing fraction at burnout. However Ellion (*9*) pointed out from a simple bubble wall heat balance that the average bubble growth rate in nucleate boiling should decrease continuously with increasing system pressure, partly because of the increased volumetric latent heat content of the vapor and partly because of the decrease in the liquid superheat. The trend has been confirmed experimentally (*10*) over a limited range of pressures (0 to 25 lb./sq. in. gauge) for several liquids. The shape of the maximum heat flux curves therefore implies that the stirring effect of the bubbles cannot be the dominant mechanism for heat transfer, at least up to a reduced pressure of about 0.5. Significantly it may be noted that the volumetric latent heat content of the vapor in general has a maximum in the range of reduced pressures of 0.3 to 0.7. Several semiempirical correlations have been proposed (*11* to *14*), where the maximum heat flux in nucleate boiling is proportional, or nearly proportional, to the volumetric latent heat content of the vapor.

One can make the observed shape of the maximum heat flux vs. reduced pressure curve more understandable by a simple calculation. If, as the bubbles become closely packed, most of the heat transfer to the bulk liquid occurs from the tops of the bubbles rather than by convection of the liquid between the bubbles, it follows that substantial vapor flow occurs within the bubble owing to simultaneous vaporization of superheated liquid near the base of the bubble and the condensation of vapor at the cooler upper walls of the bubble. This means that the average temperature of the evaporative surfaces of the bubbles T_0 is greater than the average temperature of the condensing surfaces T_1. The problem may be idealized by considering the steady state, one-dimensional flow of vapor between two liquid surfaces held at different temperatures T_0 and T_1. The solution, which has been given by Plesset (*15*), is

$$q_v = F \epsilon \lambda \rho_{0v} \left(\frac{R_G T_0}{2\pi M}\right)^{1/2} \left[1 + \left(\frac{\rho_{1v}}{\rho_{0v}}\right)\left(\frac{T_1}{T_0}\right)\right]\left[1 + \left(\frac{T_1}{T_0}\right)^{1/2}\right]^{-1} \qquad (2)$$

In general, because of the high heat transfer rates from the tops of the bubbles, T_1 is appreciably less than the saturation temperature T_s in order to maintain the driving force for condensation. Apart from the subcooling of the polar regions of the bubbles one may expect that there will be appreciable amounts of superheated liquid at the base of the bubbles. To understand this in more detail it is necessary to consider the mechanics of liquid flow near the base of a bubble which is growing on the surface of a heated solid. Because of the no-slip condition at the solid surface the liquid cannot be directly displaced from the solid as the bubble expands. A thin film of superheated liquid remains in contact with the solid surface and is not displaced by the advance of the bubble wall. This film continuously evaporates at the base of the bubble.

One now proceeds to make some illustrative calculations of the maximum heat flux as a function of the reduced pressure for several liquids, as predicted by Equation (2). Although, as noted above, $T_1 < T_s$, in order to demonstrate the importance of the evaporation of superheated liquid at the base of the bubbles one shall assume $T_1 = T_s$ and that $T_0 = T_w$. Cichelli and Bonilla (*7*) have measured the wall superheat and the heat flux at maximum pool nucleate boiling rates for five organic liquids. Their data for the wall superheat are closely approximated by an expression of the form

$$T_w - T_s = -45 \log_{10} p_r \qquad (3)$$

Equations (2) and (3) were used to calculate q_v for three organic liquids. The accommodation coefficient ranges from about 0.2 for polar organic compounds, such as ethanol, to approximately 1.0 for nonpolar compounds, such as pentane (*16*). The product $F\epsilon$ has been used as a scaling factor in determining the curves shown in Figure 1. It will be seen that the shape of the calculated heat flux curve is quite similar to those of the experimental curves. Appropriate values of $F\epsilon$ are less than 10^{-2}. The corresponding values of F are an order of magnitude less than the critical packing fraction, which shows that even if T_0 is considerably less than T_w, this mechanism will still dominate.

These results have some interesting and important consequences. First they support the turbulent heat transfer hypothesis previously advanced (*17*) for bubble growth and collapse in subcooled nucleate boiling. In highly subcooled boiling the radius-time curve is symmetrical, the growth and collapse portions of the curve being mirror images of each other. This implies that the liquid film around the bubble cannot be stagnant, since the reversibility of the heat flow process would violate thermodynamic principles. It was shown, by comparison of experimental data with the Rayleigh solution for bubble growth and collapse, that the bubbles grow and collapse under an approximately constant pressure difference during their entire visible lifetime. To account for this constant restraining pressure it was postulated (*17*) that the film of superheated liquid next to the heating surface is considerably thinner than the maximum bubble radius. Hence the bubble begins its rapid growth while entirely submerged in the thin superheated layer but begins to burst out of this layer about the time it becomes visible. The bubble is therefore growing fastest when first observed, in agreement with experiment. This is equivalent to the growth and collapse of a cavitation bubble, which receives its original impulse from the conversion of some of the superheat energy to mechanical energy. It was further postulated, because of the high rates of turbulent heat transfer at the condensing surfaces of the bubbles, that the vapor within the bubble was not in equilibrium with the liquid at the bubble wall. The pressure within the bubble is intermediate between the saturation vapor pressures of the liquid at the evaporative and condensing surfaces of the bubble. It was proposed that the observed constant difference between the pressure within the bubble and the pressure of the surroundings represents the driving force for condensation at the bubble walls necessary to maintain a relatively constant heat flux from the bubble to the liquid.

With these hypotheses used, the variation in initial kinetic energy imparted to the bubble and in the restraining pressure during bubble growth and collapse with the bulk liquid velocity and temperature and with the wall heat flux were predicted qualitatively for the data of Gunther (*1*). This picture of the bubble dynamics in subcooled nucleate boiling is now further reinforced by high turbulent heat transfer coefficients from the bubble surface (*5*) and appreciable latent heat transport.

The corresponding conclusions cannot as yet be drawn for saturated boiling, partly because of the greater experimental difficulty in obtaining bubble populations and partly because of the more complex flow pattern resulting from bubble detachment and entrainment of superheated liquid. However the results discussed above seem to argue strongly that the thermal boundary-layer thickness cannot be much larger than the maximum bubble radius, since in this event latent heat transport would be negligible. One may also note the observation of Griffith (*18*), which is in agreement with the unpublished observation of Mikesell (*19*), that in saturated boiling the bubbles are detached by the inertia of the surrounding liquid, which pulls the bubble off the surface when the bubble growth rate slows down suddenly.

Finally one may note a rather instructive analogy to the increase in heat transfer coefficient in nucleate boiling as compared with forced convection heat transfer at the same bulk liquid velocity. This is similar to the large increase in heat transfer coefficient in drop-by-drop condensation of vapors as compared with noncondensing heat transfer at the same gas stream velocity (*21*). In this case the liquid droplets grow slowly, so they cannot be considered to agitate the vapor near the wall very much (at least in comparison with the agitation required to obtain equal heat transfer in a noncondensing system). Eucken (*22*) has attributed the increased heat transfer rate to a reduction of the thickness of the layer of supercooled vapor near the surface by diffusion into the droplets, which is directly analogous to the latent heat transport mechanism proposed above.

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NOTATION

F = time-average fraction of the surface covered by bubbles
M = molecular weight of vapor
p_r = reduced pressure of the system
q_v = latent heat flux
R_a = average bubble wall velocity
R_G = gas constant
T_s = saturation temperature
T_w = average wall temperature
T_0 = average temperature of bubble evaporative surfaces
T_1 = average temperature of bubble condensing surfaces
u'_b = rms velocity induced by array of growing and collapsing bubbles
ϵ = accommodation coefficient for evaporation or condensation
λ = latent heat of vaporization
ρ_{0v} = equilibrium vapor density at liquid temperature T_0
ρ_{1v} = equilibrium vapor density at liquid temperature T_1

LITERATURE CITED

1. Gunther, F. C., *Prog. Rept. 4-75*, Jet Propulsion Laboratory, Pasadena, California (1950).
2. ———, and F. Kreith, *Prog. Rept. 4-120*, Jet Propulsion Laboratory, Pasadena, California (1950).
3. Rohsenow, W. M., and J. A. Clark, *Trans. Am. Soc. Mech. Engrs.*, **73**, 609 (1951).
4. Mixon, F. O., Jr., W. Y. Chon, and K. O. Beatty, Jr., *Chem. Eng. Progr. Symposium Ser. No. 30*, **56**, 75 (1960).
5. Bankoff, S. G., and J. P. Mason, *A.I.Ch.E. Journal*, **8**, No. 1, p. 30 (1962).
6. Kazakova, E. A., in "Problems of Heat Transfer with Change of Phase," Gosenergoizdat, Moscow (1953).
7. Cichelli, M. T., and C. F. Bonilla, *Trans. Am. Inst. Chem. Engrs.*, **41**, 755-787 (1945).
8. Bankoff, S. G., *Chem. Eng. Progr. Symposium Ser. No. 32*, **57**, 156-163 and 164-172 (1961).
9. Ellion, M. E., *Memo. 20-88*, Jet Propulsion Laboratory, Pasadena, California (1954).
10. Staniszewski, B. E., *Tech. Rept. 16, DSR Proj. 7-7673*, Mass. Inst. Technol., Cambridge, Massachusetts (1959).
11. Borishanskii, V. M., *Zhur. Tekh. Fiz.*, **25**, 252 (1956).
12. Rohsenow, W. M., and P. Griffith, *Trans. Am. Soc. Mech. Engrs.*, **74**, 1969 (1952).
13. Forster, H. K., and N. Zuber, *A.I.Ch.E. Journal*, **1**, 531 (1955).
14. Levy, S., *J. Heat Transfer*, **81**, Series C, 37 (1959).
15. Plesset, M. S., *J. Chem. Phys.*, **20**, 790-793 (1952).
16. Penner, S. S., *J. Phys. Chem.*, **56**, 475 (1952).
17. Bankoff, S. G., and R. D. Mikesell, *Chem. Eng. Progr. Symposium Ser. No. 29*, **55**, 95 (1959).
18. Griffith, P., *Tech. Rept. No. 8, D. I. C.*, Mass. Inst. Technol., Cambridge, Massachusetts (1956).
19. Mikesell, R. D., M.S. thesis, Rose Polytechnic Institute, Terre Haute, Indiana (1957).
20. Bankoff, S. G., W. J. Colahan, Jr., and D. R. Bartz, *Memo. 20-137*, Jet Propulsion Laboratory, Pasadena, California (1956).
21. Jakob, Max, "Heat Transfer," Vol. 1, Wiley, New York (1949).
22. Eucken, A., *Naturwissenschaften*, **25**, 209 (1937).

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